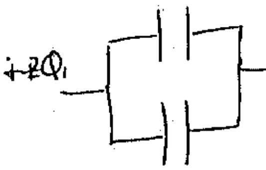
PHYS1231 Solution

Q1) A) [P1221+1231]

a) $E_1 = \frac{1}{2}C_1V_1^2 = \frac{1}{2}CV^2$; $E_2 = \frac{1}{2}C_2V_2^2 = \frac{3}{2}CV^2$

$\therefore E_T = E_1 + E_2 = 2CV^2$

b) 

$Q_1 = C_1V = CV$; $Q_2 = C_2V = 3CV = 3Q_1$

$\therefore Q_T = 3Q_1 - Q_1 = 2Q_1$

In equilibrium, charge distributes in ratio 1:3, to give the same voltage across each.

$\therefore Q_1' = \frac{1}{2}Q_1 = \frac{1}{2}CV$; $Q_2' = \frac{3}{2}Q_1 = \frac{3}{2}CV$.

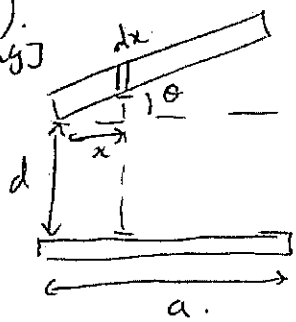
c). $V' = \frac{Q_1'}{C_1} = \frac{Q_2'}{C_2} = \frac{1}{2}V$ [same for each]

d). $E_T = \frac{1}{2}C_1V'^2 + \frac{1}{2}C_2V'^2 = \frac{1}{2}C\frac{V^2}{4} + \frac{1}{2}3C\cdot\frac{V^2}{4} = \frac{1}{2}CV^2$.

e). Large current would have moved

$\rightarrow$ bang - heat, sound, light.

B). [1231 only]



Break into strips of width dx.

$\rightarrow$ approx. a parallel plate cap.

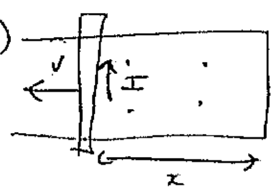
$$dC = \frac{\varepsilon_0 A}{d'} = \frac{\varepsilon_0 \cdot a\,dx}{d + x\theta}$$

Total capacitance of the combo - add in parallel. θ is small!

$$\rightarrow C = \int dC = \varepsilon_0 a \int_0^a \frac{1}{d + x\theta}\,dx \cong \frac{\varepsilon_0 a}{d}\int_0^x \left[1 - \frac{x\theta}{d}\right]dx$$

$$C = \frac{\varepsilon_0 a^2}{d}\left[1 - \frac{a\theta}{2d}\right].$$

Q2) A) [1231 only].

a) 

$$\mathcal{E} = -\frac{d\phi_B}{dt} = -\frac{d}{dt}(B\ell x) = -BLv.$$

$$\therefore I = \frac{BLv}{R} = \frac{1.18 \times 0.108 \times 4.86}{0.415} = 1.49A.$$

current clockwise to oppose motion.

b). $P = I^2R = (1.49)^2 \times 0.415 = 0.9213\,W$

$= \frac{B^2L^2v^2}{R}$.

c). $F = BI\ell = \frac{B^2L^2v}{R} = $ ~~$1.18^2 \times 0.108^2$~~ $1.18 \times 1.49 \times 0.108$

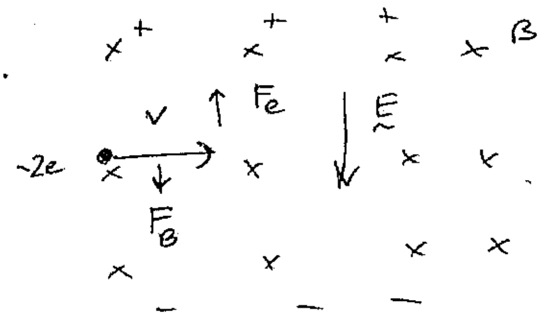
$\quad = 0.190\,N$

(1)

d). $P = F \cdot v = 0.190 \times 4.86 = 0.92 W.$

$= \frac{B^2 L^2 v^2}{R}.$

B).



$qE = qvB$

$E = vB$

$= 5.4 \times 10^6 \times 0.52$

$= 2.8 \times 10^6 \ Vm^{-1}.$

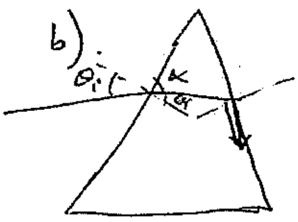
down the page

Q3 a). $\theta_c = \sin^{-1}\left(\frac{1}{1.58}\right) = 39.3^\circ.$

[1221 + 1231]

$\rightarrow \quad \phi = 90 - 39.3 = 50.7^\circ.$

b).



$\alpha = 180^\circ - 78 - 50.7 = 51.3$

$\Rightarrow \theta_r = 38.7^\circ.$

$n_1 \sin\theta_i = n_2 \sin\theta_r$

$\Rightarrow \theta_i = \sin^{-1}(1.58 \times \sin 38.7)$

$= 81.1^\circ$

$\Rightarrow \theta = 8.9^\circ.$

c). angle $> \theta$ angle $< \theta$

Q4) a) $I(\theta) = 4 I_0 \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right) \left[\frac{\sin\left(\frac{\pi a \sin\theta}{\lambda}\right)}{\left(\frac{\pi a \sin\theta}{\lambda}\right)} \right]^2$

~~[1221+12~~

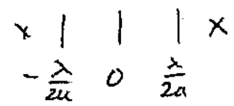
[1231 only]

double-slit   single-slit.

b). $d = 2a \rightarrow$ double-slit fringe spacing, $\theta_d \sim \frac{\lambda}{d} \sim \frac{\lambda}{2a}.$

single-slit ~~spacing~~ minimum $\theta_s \sim \frac{\lambda}{a}$

$\Rightarrow$



~~$\Rightarrow 8$~~

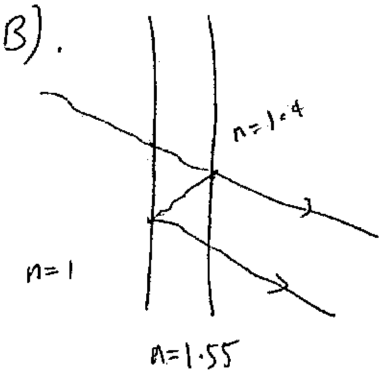
3 fringes.

c). $d = a \rightarrow I(\theta) = 4 I_0 \cos^2\left(\frac{\pi a \sin\theta}{\lambda}\right) \cdot \sin^2\left(\frac{\pi a \sin\theta}{\lambda}\right) \cdot \left[\frac{1}{\left(\frac{\pi a \sin\theta}{\lambda}\right)}\right]^2$

but $\cancel{2}\sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$

$$\therefore \quad I(\theta) = 4I_0 \cdot \left[\frac{\sin\left(\frac{2\pi a \sin\theta}{\lambda}\right)}{\left(\frac{2\pi a \sin\theta}{\lambda}\right)} \right]^2$$

B).



No phase change either reflection.

$\therefore \quad \Delta OPL = 2nt$

For constructive interference:

$\Delta OPL = m\lambda$

$\Rightarrow \quad 2nt = m\lambda.$

$t = \frac{m\lambda}{2n} = \frac{1 \times 525\,nm}{2 \times 1.55} = 169\,nm$ (minimum, $m=1$).

Q5) a) $f_0 = \phi/h = \frac{5.32 \times 1.6\times10^{-19}}{6.626\times10^{-34}} = 1.28\times10^{15}\,Hz.$

[1221+ 1231]

b). large ϕ, ~~only~~ photons with higher energy

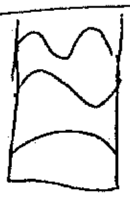
Then $hf_0 = \phi$ can eject e^-, and charge satellite.

lowers proportion of light ~~that~~ can cause charge.

c). Approx. satellite as conducting box

$\rightarrow$ no. field inside conductor

Q6).



[1271 only]

a) $\Rightarrow$ stable states $\quad n\frac{\lambda}{2} = L \rightarrow \lambda = \frac{2L}{n}.$

$\therefore \quad p = \frac{h}{\lambda} = \frac{nh}{2L} \rightarrow E = \frac{n^2 h^2}{8mL^2}$

for $n=15$, $\quad E = \frac{15^2 \times (6.626\times10^{-34})^2}{8 \times 9.1\times10^{-31} \times (98.5\times10^{-12})^2} = 8.74\,keV.$

b). $\Delta p \sim \frac{\hbar}{2\Delta x} \approx \frac{6.626\times10^{-34}}{4\pi \times 98.5\times10^{-12}} \sim 5.41\times10^{-25}\,Ns$

c) $\Delta x \sim 98.5\,pm.$

Q6). a) Bohr : I) discrete set of orbits stable

[1221 only] II) $L = mvr = n\hbar$ for stable orbit

III) Photons emitted when change orbit.

$$hf = E_{n_1} - E_{n_2}$$

b) $E_T = \frac{1}{2}mv^2 + -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$ but $\frac{mv^2}{r} = \frac{e^2}{4\pi\varepsilon_0 r^2}$

$$\rightarrow E_T = -\frac{1}{8\pi\varepsilon_0}\frac{e^2}{r}.$$

Now, $(mvr)^2 = (n\hbar)^2 = \frac{e^2 rm}{4\pi\varepsilon_0}$.

$$\Rightarrow E_n = -\frac{e^2}{8\pi\varepsilon_0} \cdot \frac{e^2 m 4\pi^2}{h^2 \cdot n^2} = -\frac{me^4}{8\varepsilon_0 h^2}\frac{1}{n^2}$$

c) line spectra of hydrogen → Rydberg eqn

[PHYS1231 only]

Q7) a) n → principal / radial → energy state

ℓ → angular momentum / (magnitude)

m_ℓ → projection of L → magnetic moment

b). $|\psi|^2$ → probability per unit volume

→ volume of spherical shell from r to $r+dr$ → $4\pi r^2 dr$

$\therefore$ $P_r(r)\,dr = |\psi(r)|^2 4\pi r^2 dr$.

→ prob. e^- between r and $r+dr$

c) $Prob = \int_0^{1.5a_0} \frac{1}{\pi a_0^3} \cdot 4\pi r^2 e^{-2r/a_0}\,dr$. let $u = r/a_0 \rightarrow du = \frac{1}{a_0}dr$

$$\therefore Prob = \int_0^{1.5} 4 \cdot u^2 e^{-2u}\,du = -2u^2 e^{-2u} + 4\int_0^{1.5} u e^{-2u}\,du$$

$$= -2u^2e^{-2u} - 2ue^{-2u} + 2\int_0^{1.5} e^{-2u}\,du$$

$$= \left[-2u^2e^{-2u} - 2ue^{-2u} - e^{-2u}\right]_0^{1.5}$$

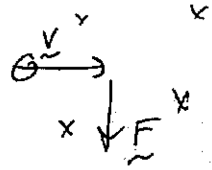
$$\simeq 0.5772.$$

PHYS1221 (Extra)

Q2) [1221 only]

a) $qV = \frac{1}{2}Mv^2 \rightarrow v = \sqrt{\frac{2qV}{M}} = \sqrt{\frac{2\times 2\times 1.6\times10^{-19}\times 105\times10^{3}}{238\times 1.67\times10^{-27}}} \simeq 4.2\times10^{5}\,ms^{-1}$

b)

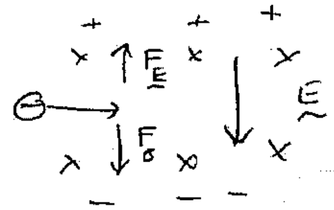


$F = qvB = 2\times1.6\times10^{-19}\times 4.2\times10^{5}\times 0.52$

$\simeq 7.0\times10^{-14}N$. (down the page).

c). $qvB = \frac{Mv^2}{r} \rightarrow r = \frac{Mv}{qB} = \frac{238\times1.67\times10^{-27}\times4.2\times10^{5}}{2\times1.6\times10^{-19}\times0.52} \simeq 1.0m$.

d)



$q\underset{\sim}{E} = qvB \rightarrow E = vB$

$= 4.2\times10^{5}\times0.52$

$= 2.2\times10^{5}\ Vm^{-1}$

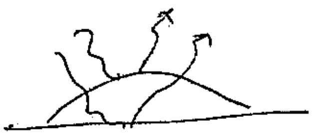
down the page.

Q3) d). Brewster angles: $\theta_{p1} = \tan^{-1}(1.55) \simeq 57.7°$

and $\theta_{p2} = \tan^{-1}\left(\frac{1}{1.55}\right) = 32.3°$

Neither $\theta_r \simeq \theta_{p1}'$ or $\theta_i \simeq \theta_{p2}$, so it should be unpolarised.

Q4).



a) phase change on both reflection

→ outer edge bright.

→ both waves in phase

b). $\Delta OPL = m\lambda$ (for constructive interference).

$\rightarrow \Delta OPL = 2n_{oil}t = 3\lambda \rightarrow t = \frac{3\lambda}{2n_{oil}} = \frac{3\times350nm}{2\times1.2} \simeq 440nm$.

c) lose coherence as path length increases.

Q5) d). $(\text{photons}/m^2) = \frac{I}{hf} = \frac{1.38\times10^{-3}}{6.626\times10^{-34}\times1.28\times10^{15}} = 1.63\times10^{15}$ photons/m³.